2025

Four Year Under Graduate Programme (FYUGP)

4th Semester Examination (Under NEP)
(Session 2023-2024)

MATHEMATICS (Major)

Paper Code : MTMMJ MC-06

(Mechanics)

Full Marks : 40 Time : Two Hours

The figures in the margin indicate full marks. Candidates are required to give their answers in their own words as far as practicable.

Group - A

(Marks : 10)

1. Answer any *five* questions : 2×5=10

(a) The position of a moving particle at time t is given by $x = a\cos nt, y = a\sin nt$. Find the velocity and acceleration of the particle.

(b) A particle moves along a plane curve. Prove that the areal velocity is $\frac{1}{2}r^2\dot{\theta}$ and $r^2\dot{\theta} = vp$.

(c) If P be the momentum of a particle of mass m, then find the kinetic energy.

(d) A particle of mass m moves in a straight line under an acceleration mn^2x towards a point O on the line, where x is the distance from O. Show that, if $x = a$ and $\dot{x} = u$ when $t = 0$, then at time t, $x = a\cos nt + \frac{u}{n}\sin nt$.

(e) The velocity of a particle distant x from a fixed point O is given by $v^2 = a - bx^2$, where a and b are constants. Show that the motion is simple harmonic and determine its period.

(f) A particle moves with a simple harmonic motion, its position of rest being at a distance a from the centre. Find, by the principle of energy, the velocity at the centre.

(g) A particle describes a parabola $p^2 = ar$ under a force which is always directed towards its focus. Find the law of force.

Group - B

(Marks : 30)

Answer any *six* questions : 5×6=30

2. Two bodies m_1 and m_2 are attached to the lower end of an elastic string whose upper end is fixed and are hung at rest; m_2 falls off. Show that the distance of m_1 from the upper end of the string at time t is $a + b + c\cos\sqrt{g/b}\,t$.

3. An engine working at a constant rate H draws a load of mass M against a resistance R. Show that maximum speed attained is $\frac{H}{R}$ and time taken to attain half this speed is

$$\frac{MH}{R^2}\left[\log 2 - \frac{1}{2}\right].$$

4. A particle moves under a force $m\mu\left\{3au^4 - 2\left(a^2 - b^2\right)u^5\right\}$ where $a > b$ and is projected from an apse at a distance $(a + b)$ with velocity $\sqrt{\mu}/(a+b)$. Show that its orbit is $r = a + b\cos\theta$.

5. Two particles of masses m_1 and m_2 moving in coplanar parabolas round the sun, collide at right angles and coalesce, when their common distance from the sun is R. Show that the subsequent path of the combined particle is an ellipse of major axis

$$\frac{(m_1 + m_2)^2 R}{2m_1m_2}$$

6. If the nearly circular orbit of a particle is $p^2\left(a^{m-2} - r^{m-2}\right) = b^m$, then show that the apsidal angle is nearly $\frac{\pi}{\sqrt{m}}$.

7. A particle descends down a rough circular tube starting from rest at an extremity of horizontal diameter. If it stops at the lowest point, show that the coefficient of friction satisfies the equation

$3\mu e^{-\mu\pi} + 2\mu^2 - 1 = 0.$

8. A particle of mass m is projected vertically upwards under gravity, the resistance of the air being mk times the velocity. Show that the greatest height attained by the particle is $\frac{V^2}{g}\left[\lambda - \log_e(1+\lambda)\right]$, where V is the terminal velocity of the particle and λV is the initial velocity.

9. A falling raindrop has its radius uniformly increased at the rate λ by gathering moisture. If it be given a horizontal velocity $2\lambda a$, then show that it will describe a hyperbola $x^2 + \frac{8\lambda^2 a}{g}xy = \frac{8\lambda^2 a^2}{g}y.$

10. A straight smooth tube revolves with constant angular velocity ω in a horizontal plane about one extremity which is fixed. If at zero time a particle inside it be at a distance 'a' from a fixed end and moving with constant velocity V along the tube, then show that its distance at time t is $a\cosh\omega t + \frac{V}{\omega}\sinh\omega t$.

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4th Semester Examination (Under NEP)

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MATHEMATICS (Major)

Paper Code : MTMMJ MC-07

(Numerical Methods and C Programming Language)

Full Marks : 40 Time : Two Hours

The figures in the margin indicate full marks. Candidates are required to give their answers in their own words as far as practicable.

Group - A

(Marks : 10)

1. Answer any *five* questions : 2×5=10

(a) Find absolute error and relative error in the approximate representation of $\frac{5}{6}$ by 0.8333.

(b) In order to find the real root of $x^3 - x - 1 = 0$ near $x = 1$, which of the following iteration functions give convergent sequences :

$$x = x^3 - 1, x = \frac{x+1}{x^2}, x = \sqrt{\frac{x+1}{x}} ?$$

P.T.O.

(c) Find the value of $\left(\frac{\Delta^2}{E}\right)x^2$.

(d) Use Newton-Raphson method to find an iteration formula for finding the real cube root of a positive real number a.

(e) What are the degrees of precision of Trapezoidal and Simpson's one-third rule? Justify your answer. 1+1

(f) What is the difference between 'while' loop and 'do-while' loop in C language?

(g) Write a program in C to print "University of Gour Banga". Explain your code.

Group - B

(Marks : 30)

Answer any *six* questions : 5×6=30

2. Explain Newton-Raphson method for computing a simple real root of an equation $f(x)=0$. When does the method fail? 4+1

3. Deduce composite Simpson's one-third rule for numerical integration from Newton's forward interpolation formula. Interpret it geometrically. 4+1

4. Solve the following system of linear equations, correct to four decimal places, by Gauss-Seidel method :

$$27x + 6y - z = 85$$
$$6x + 15y + 2z = 72$$
$$x + y + 54z = 110$$

5. Explain the method of Euler to solve the differential equation $\frac{dy}{dx} = f(x,y),\ y(a) = y_0,\ x \in [a,b]$.

Exhibit its geometrical illustration. 3+2

6. What is interpolation? Establish Lagrange's interpolation formula. 1+4

7. Using Regula-Falsi method, compute a real root of the equation $3x - \cos x - 1 = 0$, correct up to four significant figures.

8. Prove that $\Delta^n x^{(n)} = n!h^n$ and $\Delta^{n+1} x^{(n)} = 0$, where symbols have their usual meaning.

9. Write a program in C to print all the prime numbers from 1 to 100.

10. Write a program in C to find the sum

$$S = \sum_{m=1}^{20} \sum_{n=1}^{20} \frac{1}{m^2 + n^2}$$